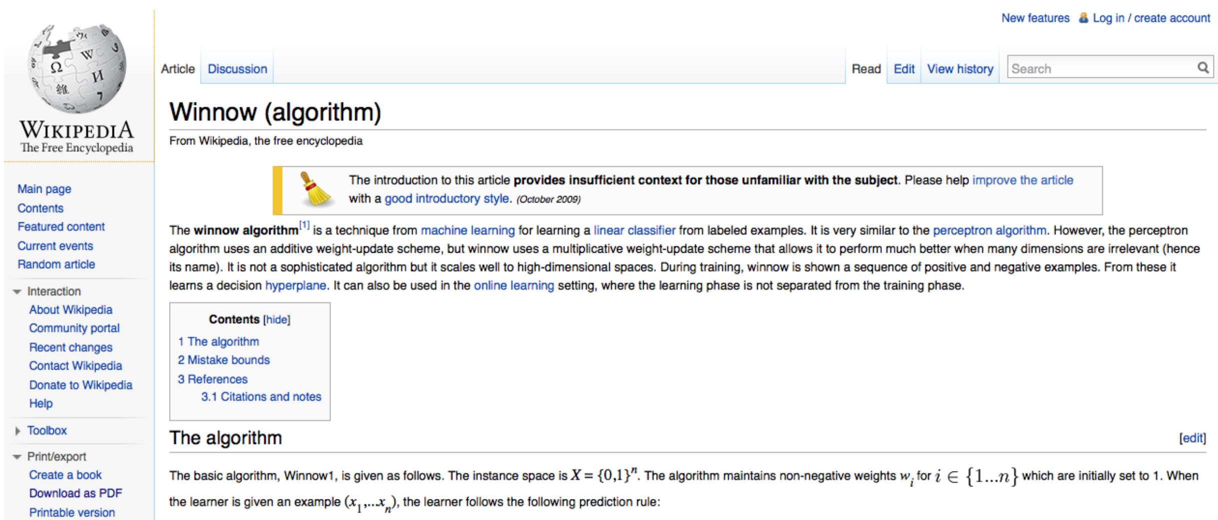
Attribution

- These slides were prepared for the New Jersey Governor’s School course “The Math Behind the Machine” taught in the summer of 2012 by Grant Schoenebeck
- Large parts of these slides were copied or modified from the previous years’ courses given by Troy Lee in 2010.

Learning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Winnow (algorithm)

From Wikipedia, the free encyclopedia

The introduction to this article **provides insufficient context for those unfamiliar with the subject**. Please help improve the article with a good introductory style. *(October 2009)*

The **winnow algorithm**[1] is a technique from machine learning for learning a linear classifier from labeled examples. It is very similar to the perceptron algorithm. However, the perceptron algorithm uses an additive weight-update scheme, but winnow uses a multiplicative weight-update scheme that allows it to perform much better when many dimensions are irrelevant (hence its name). It is not a sophisticated algorithm but it scales well to high-dimensional spaces. During training, winnow is shown a sequence of positive and negative examples. From these it learns a decision hyperplane. It can also be used in the online learning setting, where the learning phase is not separated from the training phase.

Contents [hide]

The algorithm [edit]

The basic algorithm, Winnow1, is given as follows. The instance space is $X = \{0,1\}^n$. The algorithm maintains non-negative weights w_i for $i \in \{1 \ldots n\}$ which are initially set to 1. When the learner is given an example $(x_1, \ldots x_n)$, the learner follows the following prediction rule:

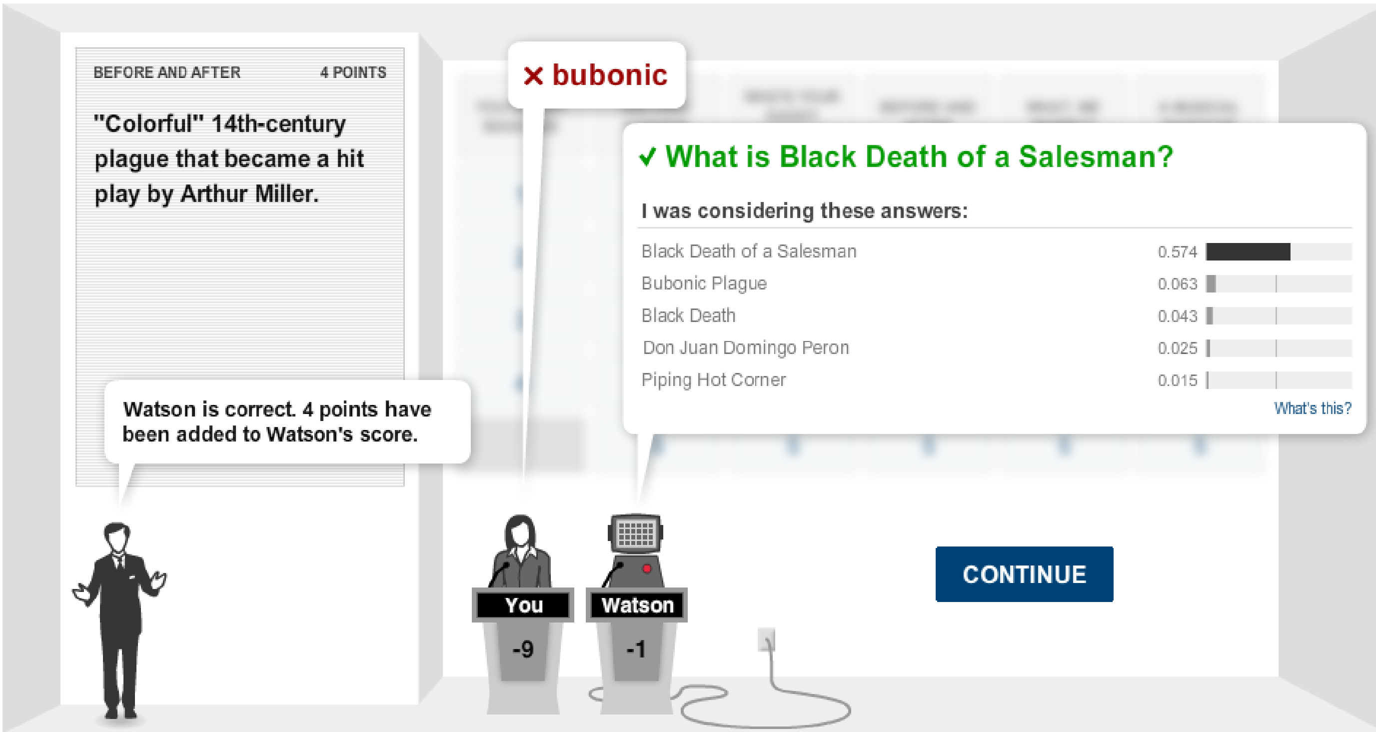
Is our computers learning?

- What does it mean for a computer to “learn”?
  - Prediction with low error
  - Compression
- Cannot “learn” random functions!
  - Unpredictable
  - Uncompressable
- Must assume function “nice” or bias toward certain functions
- Many different scenarios
  - Unsupervised learning
    - Clustering
  - Supervised learning
  - Semi-supervised Learning
  - Learning and Optimizing

Examples of Computational Learning

- Spam
- Deep Blue
- Search Engines
- Datacenter Power conservation
- Image Search
- Face recognition in Facebook

Answering Questions

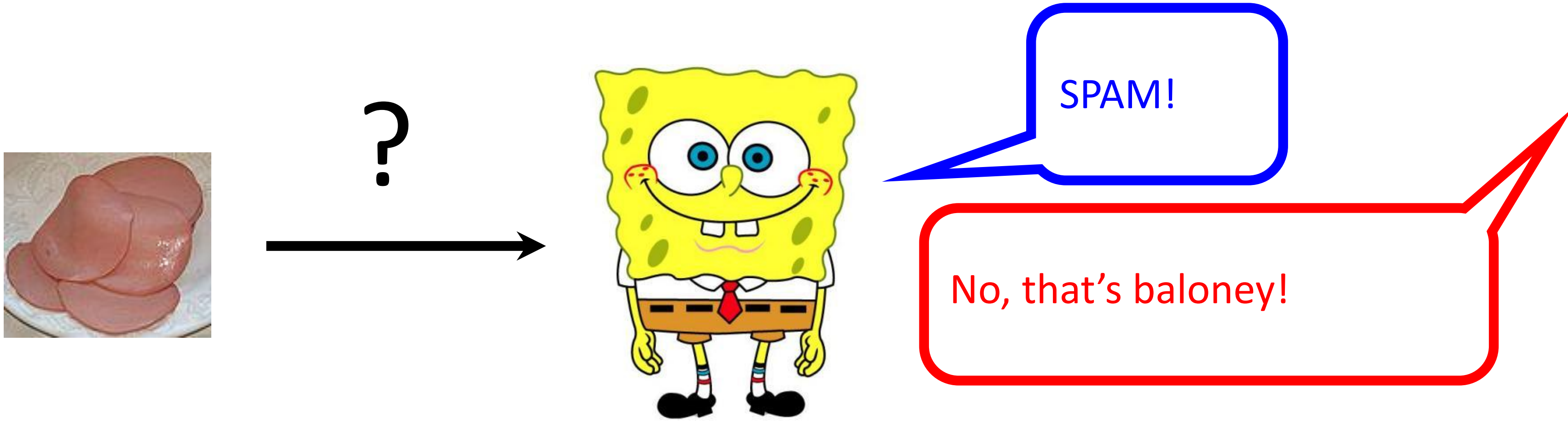


http://www.nytimes.com/2010/06/20/magazine/20Computer-t.html

Prediction

- Predict a user's rating based on previous rankings and rankings of other users.
- Recently, Netflix gave out $1,000,000 in contest to improve their ranking system by 10%.
  - Winning algorithm was a blend of many methods.
  - Surprising relevant information:
    - Ranking a movie just after watching vs. years later...

Online Learning

- Bob is given examples on the fly, and has to classify them.
- 

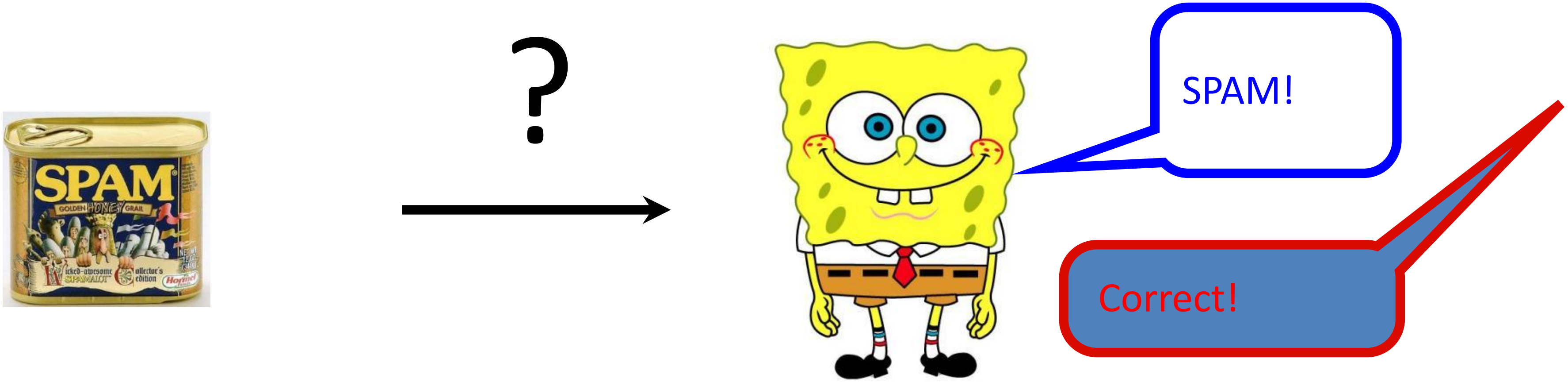
- After each example, Bob is given correct answer.

Online Learning

- Focus on the binary case: spam vs. not spam

SPAM

~~SPAM~~



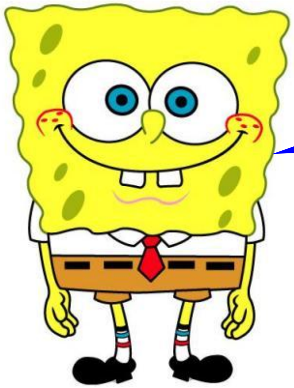
Online Learning

Focus on the binary case:
spam vs. not spam

SPAM

~~SPAM~~

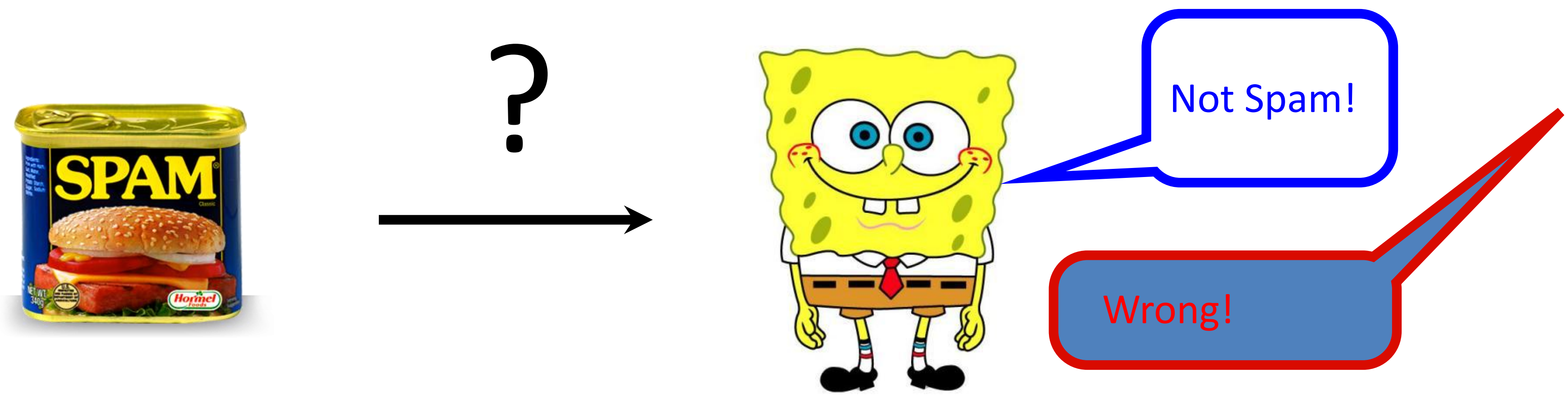
?



Not Spam!

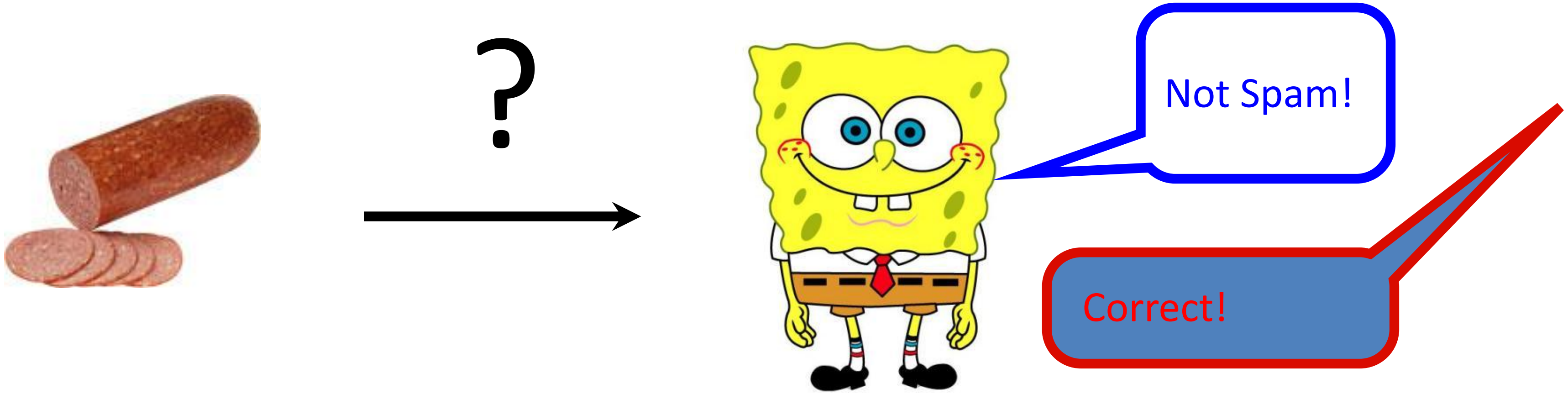
Correct!

Online Learning



Online Learning

Interested in Total Number of Mistakes



More abstractly...

Bob is learning an unknown Boolean function u(x).

– Here, u is a function on lunch meats.

– Usually, have an assumption that u comes from a known family of functions---called a concept class.

→ 0

→ 1

→ 0

Learning Disjunctions

- We will assume the unknown function u comes from a simple family: monotone disjunctions.
  - $u(x) = x_{i_1} \vee x_{i_2} \vee \cdots \vee x_{i_k}$
  - Monotone means no negations

Disjunctions

x_1 = [Aardvark]

x_2 = [Aarrghh]

$\vdots$

$x_{601,384}$ =[Zyzzyva]

A simple “bag of words” model of spam---

An email is spam if it contains one of a handful of keywords: viagra, lottery, xanax...

SPAM(x)=viagra(x) OR lottery(x) OR ... OR xanax(x)

We have variables to indicate if a word is present in email x.

n=Total # variables, k=number present in disjunction.

Basic Algorithm

Maintain a hypothesis h(x) for what the disjunction is.

Initially, $$h(x) = x_1 \vee x_2 \vee \cdots \vee x_n$$

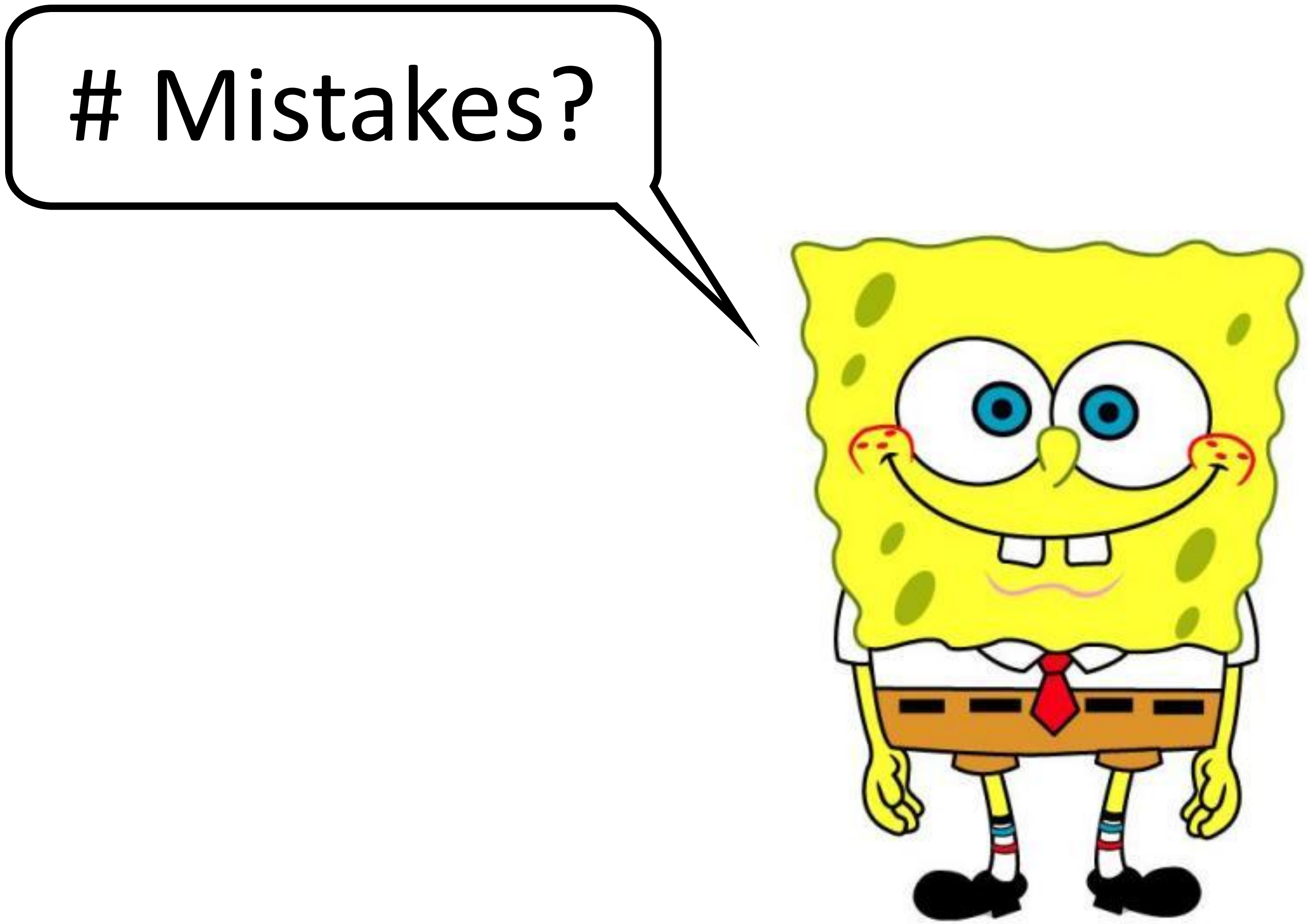
That is, we assume disjunction includes all variables.

This will label everything as spam!

Update Rule

Every time we get an answer wrong, we will modify our hypothesis, as follows:

- On false positive, i.e. SPAM(x)=0 and h(x)=1, remove all variables from h where $x_i = 1$.
- On false negative, i.e. SPAM(x)=1 and h(x)=0, output FAIL.
  - If the function we are learning is actually a monotone disjunction, false negatives never happen.
  - Variables present in hypothesis always a superset of those in SPAM function.


Mistakes?

Number of Mistakes

Claim: This algorithm makes at most n mistakes if unknown function is indeed a disjunction.

Every time we get an answer wrong, we will modify our hypothesis, as follows:

- On false positive, i.e. SPAM(x)=0 and h(x)=1, remove all variables from h where $x_i = 1$.
- On false negative, i.e. SPAM(x)=1 and h(x)=0, output FAIL.
  - If the function we are learning is actually a monotone disjunction, false negatives never happen.
  - Variables present in hypothesis always a superset of those in SPAM function.

- When we make a mistake at least one word is eliminated from hypothesis

Importance of Good Teachers...

- What happens if some example is mislabeled?
  - $u(x)=1$ but Teacher mistakenly says $u(x)=0$.
  - The weight for some variable relevant to u will be set to zero, and can never come back!
- Then we can make an unbounded number of mistakes.

Sparse Disjunctions

- This algorithm has disadvantage that the number of mistakes can depend on total number of variables.
  - In spam example, even though there are only a few keywords, we could make make mistakes on order the number of words in English!
- Ideally, would like an algorithm that depends on the number of relevant variables rather than total number of variables.

Winnow Algorithm

- Winnow- to separate the chaff from the grain
- The winnow algorithm quickly separates the irrelevant variables from the relevant ones.

Winnow Algorithm

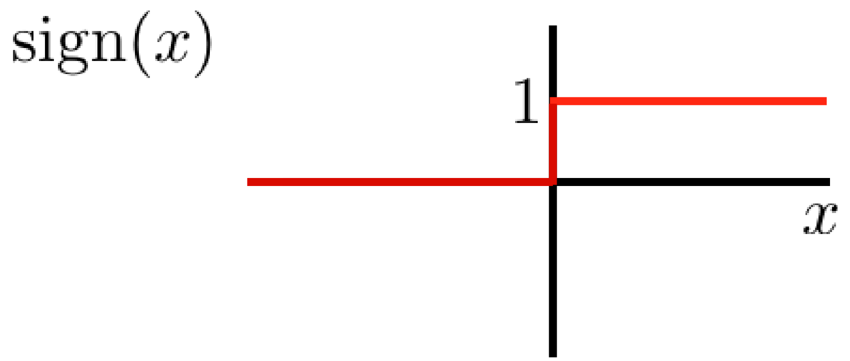
- Say the unknown function only depends on k variables out of a total of n.
  - $u(x) = x_{i_1} \vee x_{i_2} \vee \cdots \vee x_{i_k}$
  - The winnow algorithm has a mistake bound of about $k \log(n)$
  - Much improved if $k \ll n$

Linear Threshold Function

- In the winnow algorithm, Bob maintains as hypothesis a linear threshold function.

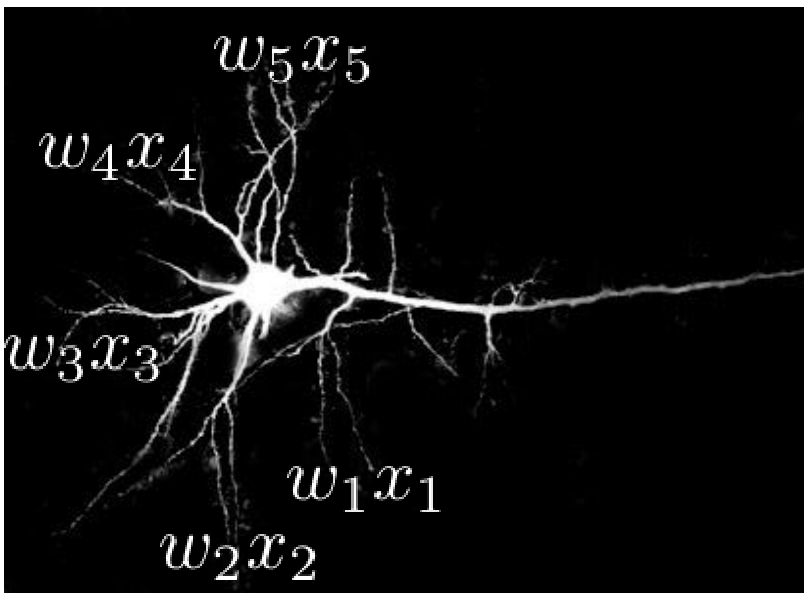
$$h(x) = \mathrm{sign}(w_1 x_1 + w_2 x_2 + \cdots + w_n x_n - \theta).$$

- Weights w_i and threshold θ are real numbers.



Linear Threshold Function

- Linear threshold functions are used as a simple model of a neuron.



$$\sum_i w_i x_i \geq \theta?$$

Linear Threshold Function

$$h(x) = \mathrm{sign}(w_1 x_1 + w_2 x_2 + \cdots + w_n x_n - \theta).$$

What can linear threshold functions do?

$$\mathrm{OR}(x) = \mathrm{sign}(x_1 + x_2 + \cdots + x_n - 1/2).$$

$$\mathrm{AND}(x) = \mathrm{sign}(x_1 + x_2 + \cdots + x_n - (n - 1/2)).$$

$$\mathrm{MAJ}(x) = \mathrm{sign}(x_1 + x_2 + \cdots + x_n - n/2)$$

Linear Threshold Function

Another sanity check: the unknown function

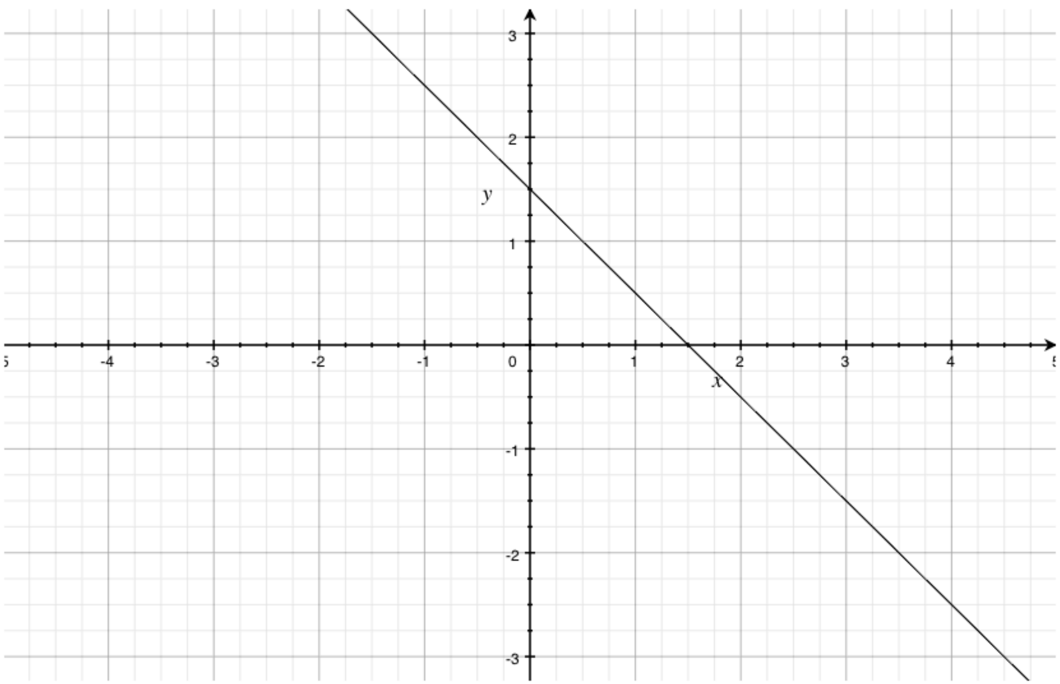
$$u(x) = x_{i_1} \vee x_{i_2} \vee \cdots \vee x_{i_k}$$

can also be expressed as a LTF.

$$\mathrm{sign}(x_{i_1} + x_{i_2} + \ldots + x_{i_k} - \tfrac{1}{2})$$

Linear Threshold Function

Linear threshold function defines a plane in space, evaluates which side of the plane a point lies on.

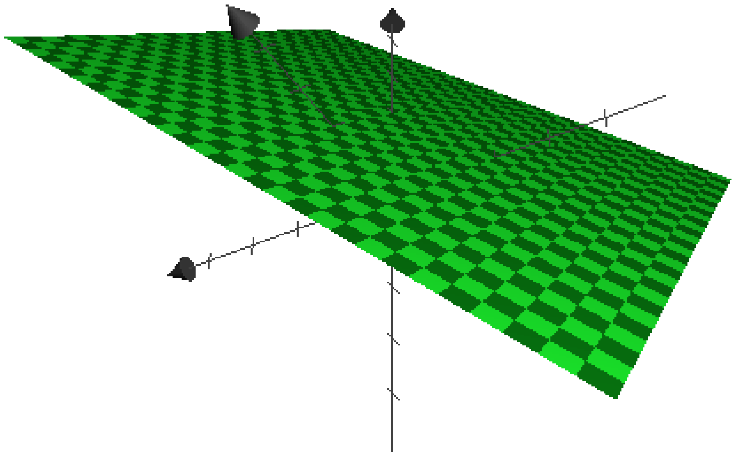


$$x + y = \frac{3}{2}$$

Linear Threshold Function

Linear threshold function defines a plane in space, evaluates which side of the plane a point lies on.

$$-x + y + z = 1$$



Linear Threshold Function

Can every function be expressed as a linear threshold function?

No, for example

$$\mathrm{PARITY}(x) = x_1 + x_2 + \cdots + x_n \mod 2$$

Note that linear threshold functions are monotone in each coordinate.

Linear Threshold Function

- In the winnow algorithm, Bob maintains as hypothesis a linear threshold function.

$$h(x) = \mathrm{sign}(w_1x_1 + w_2x_2 + \cdots + w_nx_n - \theta).$$

- Initially, all weights are set to 1 and θ=n.
- Bob guesses h(x) for the current hypothesis h.
  - If correct: stay the course
  - If wrong: update weights

Recap so far...

We want to learn a function which is a disjunction of k variables out of n possible.

$$u(x) = x_{i_1} \vee x_{i_2} \vee \cdots \vee x_{i_k}$$

Initially, we take as hypothesis the function

$$\mathrm{sign}(x_1 + x_2 + \ldots + x_n - n)$$

After each mistake we will update the weight of each variable. Threshold will stay the same.

Update Rules

In the winnow algorithm, Bob maintains as hypothesis a linear threshold function.

$$h(x) = \text{sign}(w_1 x_1 + \cdots + w_n x_n - n)$$

Update: On false positive, i.e. u(x)=0 and h(x)=1, Bob sets

$$w_i = 0 \text{ for all } i \text{ where } x_i = 1.$$

On false negatives, i.e. u(x)=1 and h(x)=0, Bob sets

$$w_i \leftarrow 2w_i \text{ for all } i \text{ where } x_i = 1.$$

Reasonable Update?

Update: On false positive, i.e. u(x)=0 and h(x)=1, Bob sets

$$w_i = 0 \text{ for all } i \text{ where } x_i = 1.$$

On false negatives, i.e. u(x)=1 and h(x)=0, Bob sets

$$w_i \leftarrow 2w_i \text{ for all } i \text{ where } x_i = 1.$$

If the unknown disjunction contains x_i , then w_i will never be set to 0.

On false negatives, give more weight to all variables which could be making the disjunction true.

Simple Observations

$$h(x) = \mathrm{sign}(w_1 x_1 + \cdots + w_n x_n - n)$$

Update: On false positive, i.e. u(x)=0 and h(x)=1, Bob sets

$$w_i = 0 \text{ for all } i \text{ where } x_i = 1.$$

On false negatives, i.e. u(x)=1 and h(x)=0, Bob sets

$$w_i \leftarrow 2w_i \text{ for all } i \text{ where } x_i = 1.$$

1) Weights always remain non-negative.

2) No weight will become larger than 2n.

Mistake Bound

Call a weight doubling a promotion, and a weight being set to 0 a demotion.

By the second observation, a weight will only be promoted at most log n times. Otherwise, becomes larger than 2n.

As there are only k relevant variables in the unknown disjunction, total number of promotions is at most k log n.

Mistake Bound

Consider when u(x)=0 and h(x)=1. Then

$$\sum_{i:x_i=1} w_i \geq n.$$

After update, all of these weights will be set to 0.

Thus the sum of all the weights will decrease by at least n after the update.

Mistake Bound

Consider when u(x)=1 and h(x)=0. Then

$$\sum_{i:x_i=1} w_i < n.$$

After update, all of these weights will be doubled.

Thus the sum of all the weights will increase by at most n after the update.

Mistake Bound

- For every demotion:
  - sum of weights decreases by at least n.
- For every promotion:
  - sum of weights increases by at most n.
- Total number of promotions is at most k log n.
- We know that the sum of the weights must remain non-negative.
- Thus number of demotions at most 1+k log n.
- If the unknown function is a k-disjunction, the total number of mistakes of the winnow algorithm is bounded by about 2k log n.
- Still suffers from errors in Teacher ☹

A Softer Version

This algorithm is harsh---once a weight is set
to zero it can never come back.

We can instead consider a softer variation:

Update: On false positive, i.e. u(x)=0 and h(x)=1, Bob sets

$$w_i \leftarrow \frac{w_i}{2} \text{ for all } i \text{ where } x_i = 1$$

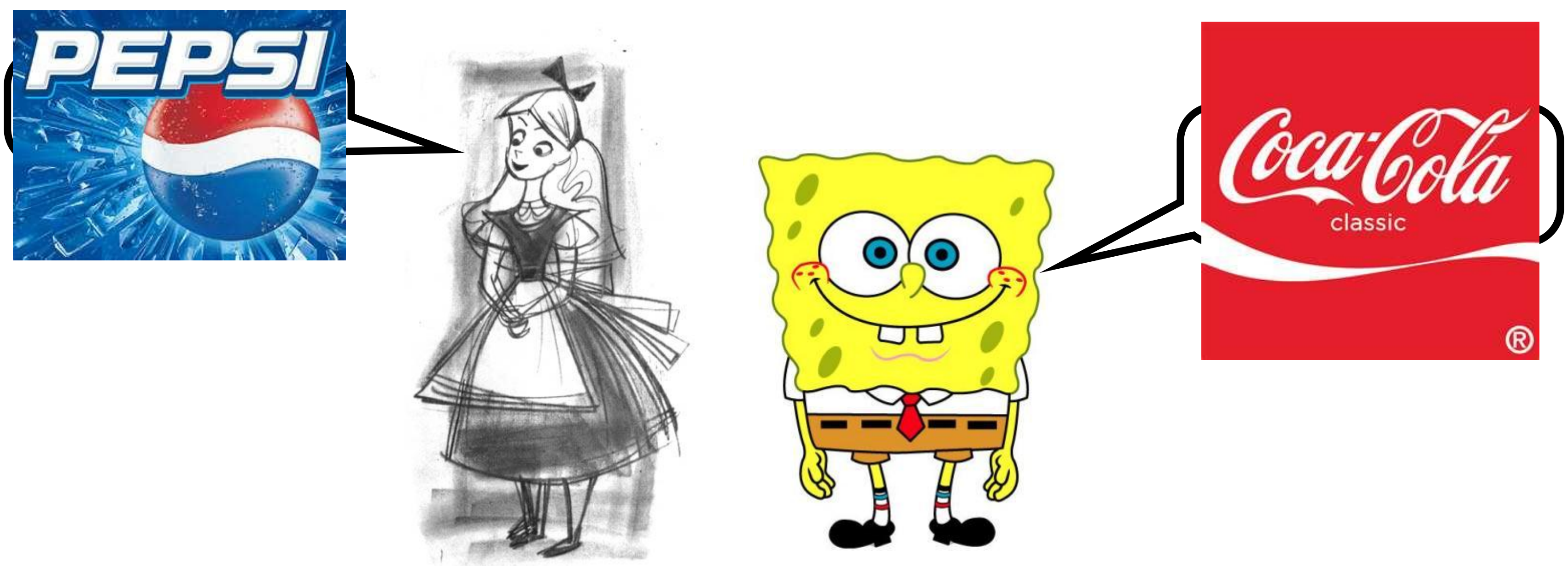
On false negatives, i.e. u(x)=1 and h(x)=0, Bob sets

$$w_i \leftarrow 2w_i \text{ for all } i \text{ where } x_i = 1.$$

Learning from Experts

Will Pepsi outperform Cocacola next month??

Everyone has a prediction!



Learning from Experts

- Whom should we listen to?
  - n experts
  - k is number of mistakes made by best expert
  - t is number of predictions
- Regret = Best Expert's Performance - Our Performance
- Can follow an amazing algorithm!
  - algorithm makes < $k + \sqrt{t \log(n)}$ mistakes
  - This is without knowing the best expert in advance!

Weighted Majority

The algorithm is very similar to Winnow.

Give a weight to each expert. Initially, set all weights equal to 1.

Predict according to the side with more weight.

After each example, update the weights:

If expert i is wrong: $w_i \leftarrow 0.99 w_i$

If expert i is right: $w_i \leftarrow w_i$

Uses of multiplicative Updates

- Weak Learners to Strong Learners
  - Adaboost
- Hard Core Sets
- Approximate solutions to linear programs
- Approximate solutions to semidefinite programs
- Find optimal strategies in zero-sum games